Microeconometrics

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July , 2009

Modern Economics

- Macroeconomics
- Microeconomics
- Financial Economics
- Econometrics

- Modern economics is to study how to allocate scarce resource in an uncertain market environment.

- Quantitative analysis is an important tool used in modern economics research.

    - Mathematical modeling

    - Empirical analysis

Econometrics

- Uses statistical tools and develops new statistical methods to analyze economic data and models.
- Is a core field of modern economics.
- It is a combination of statistics, economic theory, and mathematical modeling.
- Mainly consists of three subfields: Macroeconometrics, Financial Econometrics, and Microeconometrics
  - Macroeconometrics and financial econometrics deal with aggregate (time series) data and analyze macroeconomic and financial economic models.
  - Microeconometric analysis with individual level data and analyzes microeconomic models.

Microeconometrics

- Microeconometric analysis is the analysis of individual-level data on the economic behavior of individuals or firms.
- Analysis is usually applied to cross-section or panel data.
  - A cross-section data set refers to a data set of a large number of individuals.
  - A panel data set refers to a data set of observations for a number of individuals across time.

Why Microeonometrics?

- Greater availability of cross-section data and longitudinal survey and census data.
- Greater computing power.
- Collecting and analyzing large and complex individual level data has raises methodological and modeling issues that drive the development of microeonometrics.

Distinctive Features and Advantages of Microeconometrics

- Disaggregation makes it possible to control for individual heterogeneity.
- Discreteness and nonlinearity of response.
- More informative.

Microeconometric Modeling Approaches

- The structural approach
  - Derives the econometric model closely from economic theory.
  - The objective is to identify the deep (primitive) structural parameters that characterize individual tastes/preferences, and other underlying relationships.
  - Heavily uses economic theory to make casual inference.
- The reduced-form approach
  - Models relationships between response variables of interest conditionally on the variables that are taken as given.
  - Is usually conducted through regression analysis.
  - Does not always take into account all the causal dependences.

Information Acquisition or/and Bid Preparation: A Structural Analysis of Entry and Bidding in Timber Sale Auctions

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August , 2009

Outline of the talk

- Review relevant theoretical and empirical literature on auctions with entry;
- Set up the competing models;
- Theoretical Implications;
- Structural econometric framework;
- Bayesian estimation and model selection methods;
- Empirical Analyses.

Auctions with Entry

- Entry is important because
  - It is an integral part of the auction game;
  - It affects the competition in and outcomes of the bidding stage.
- Theoretical Studies:
  - Levin and Smith (1994, AER): entry costs include both information acquisition and bid preparation costs.
  - Samuelson (1965, EL): entry costs include only bid preparation cost.
- Empirical Studies:
  - Variants of Levin and Smith Model: Bajari and Hortacsu (2003, RJE), Athey, Levin and Seira (2004), Li (2005, JoE), Krasnokutskaya and Seim (2006), Li and Zheng (2009, ReStud).
  - Variant of Samuelson Model: Li and Zheng (2009, ReStud).
- Distinguishing the two models is important because: Different policy implications.

Model I: The Model with Information Acquisition and Bid Preparation Entry Costs

- The seller (government here) auctions a single and indivisible timber harvesting right;
- Posts binding reserve price p_0;
- N potential bidders;
- Each potential bidder is risk-neutral with a private value v of the timber;
- Each potential bidder must incur an entry cost k;
- First stage: Learns N and auction specifics and decides whether to incur k to enter;
- Second stage:
  Active bidders (n) learn v and if $v \geq p_0$, become actual bidders;
  Actual bidders (n^*) submit b without the knowledge of n.

First-Stage Entry

$$\int_{p_0}^{\overline{v}} E\pi(b, v|q^*) f(v) dv = k.$$

v: private values;

$f(\cdot)$: density of private values with support $[\underline{v}, \overline{v}]$;

k: entry cost;

b: bids;

q^*: equilibrium entry probability;

$E\pi(b, v|q^*)$: expected payoff for the actual bidder.

In equilibrium, this condition determines the equilibrium entry probability q^*.

Second-Stage Bidding

- An active bidder's objective function:
  $E\pi(b_i, v_i|q^*) = \sum_{j=1}^{N} P_B(n=j)(v_i - b_i)\Pr(b_t \leq b_i, \forall t \neq i)$ if $v_i \geq p_0$ where $P_B(n=j) = \binom{N-1}{j-1} q^{*j-1}(1-q^*)^{N-j}$.
- Important assumption:
  Active bidders do not know the number of active bidders when they bid.
- With the boundary condition $s(p_0) = p_0$, the unique symmetric increasing Bayesian-Nash equilibrium bidding strategy for an active bidder in this model is:

$$b = s(v|q^*) = v - \frac{\sum_{j=1}^{N} P_B(n=j)\int_{p_0}^{v} F(x)^{j-1}dx}{\sum_{j=1}^{N} P_B(n=j)F(v)^{j-1}}.$$

- And q^* is determined by

$$\int_{p_0}^{\overline{v}} \sum_{j=1}^{N} P_B(n=j)\int_{p_0}^{v} F(x)^{j-1}dx f(v)dv = k.$$

Model II: The Model with only Bid Preparation Entry Cost

- Each potential bidder learns N, auction specifics and private value v and decides whether to incur k to enter;
- Potential bidders with $v \geq v^*$ become actual bidders;
- n^* Actual bidders submit b.
- With the boundary condition $s(v^*) = p_0$, the unique symmetric increasing Bayesian-Nash equilibrium bidding strategy for an active bidder in this model is:

$$b = s(v) = v - \frac{\int_{v^*}^{v} F(x)^{N-1} dx}{F(v)^{N-1}} + \frac{F(v^*)^{N-1}}{F(v)^{N-1}} (p_0 - v^*) \,.$$

- And v^* is determined by

$$(v^* - p_0) \, F(v^*)^{N-1} = k.$$

Model Implications

- Proposition I: In both model I and model II, the relationship between b and N may not be monotone increasing.
- Proposition II: In both model I and model II, the relationship between W (the expected winning bid) and N may not be monotone increasing.
- Proposition III: In model I, the seller's optimal reserve price is her own value, that is, $p_0^{opt} = v_0$.
- Proposition IV: In model II, the seller's optimal reserve price is

$$p_0^{opt} = v_0 + \frac{1 - F(v^{*opt})}{f(v^{*opt})}$$

where v^{*opt} is defined implicitly in

$$v^* = v_0 + \frac{1 - F(v^*)}{f(v^*)} + \frac{k}{F(v^*)^{N-1}}.$$

Structural Econometric Framework

- $v_{i\ell} \sim f(\cdot|x_\ell, u_\ell, \beta)$ and $k_\ell \sim h(\cdot|x_\ell, u_\ell, \delta)$ for $i = 1, 2, ...n_\ell^*$ and $\ell = 1, 2, ...L$;
  - x_ℓ: observed heterogeneity;
  - u_ℓ: unobserved heterogeneity;
  - β and δ: unknown parameter vector;
  - n_ℓ^*: number of actual bidders.
  - N_ℓ: number of potential bidders.
- $f(v|x, u)$ is exponential with mean $\exp(x\beta + u)$.
- $h(k|x, u)$ is exponential with mean $\exp(x\delta + u)$.
- Distribution of u is assumed to be normal.

Sovling the Model and Implied Densities

- Solution of Model II

$$b = s(v) = v - \frac{\int_{v^*}^{v} \left\{1 - \exp[-\frac{1}{\exp(\mu_1)}x]\right\}^{N-1} dx}{\left\{1 - \exp[-\frac{1}{\exp(\mu_1)}v]\right\}^{N-1}} + \frac{\left\{1 - \exp[-\frac{1}{\exp(\mu_1)}v^*]\right\}^{N-1}}{\left\{1 - \exp[-\frac{1}{\exp(\mu_1)}v]\right\}^{N-1}} (p_0 - v^*),$$ where $\mu_1 = x\beta + u$

$$(v^* - p_0)\left\{1 - \exp[-\frac{1}{\exp(\mu_1)}v^*]\right\}^{N-1} = k.$$

- Implied densities for b and v^*

$$f(b|\mu_1) = \frac{1}{\exp(\mu_1)} \exp\left\{-\frac{1}{\exp(\mu_1)}\phi(b)\right\} |\frac{\partial \phi(b)}{\partial b}|$$

for $b \in$

$$\left[p_0, \int_{v^*}^{\infty} \frac{x(N-1)}{\exp(\mu_1)} \left\{1 - \exp[-\frac{1}{\exp(\mu_1)}x]\right\}^{N-2} \exp[-\frac{1}{\exp(\mu_1)}x]dx + p_0 \left\{1 - \exp[-\frac{1}{\exp(\mu_1)}v^*]\right\}^{N-1} \right]$$

$p(v^*|\mu_1, \Theta) = h(k|\mu_1, \Theta) \times |\frac{\partial k}{\partial v^*}| \times \mathbb{1}[p_0 \leq v^* \leq \phi(b_{n^*})]$ and $\Theta = \beta - \delta$.

Estimation Method: Bayesian

Why Bayesian?

- Classical MLE or nonparametric methods are intractable;
- Computationally efficient;
- Give finite sample properties of the resulting estimates;
- Statistically efficient according to the local asymptotic minmax criterion for standard loss functions; (parameter dependent support problem in structural auction models)
- Takes into account the unobserved heterogeneity easily.

Bayesian Estimation: Posterior

$$\pi(\beta, \Theta, \sigma^2, \{\mu_{1,\ell}, v_\ell^*\}_{\ell=1}^{L} | b, n^*)$$

$$\propto \; prior(\beta, \Theta, \sigma^2) \times \prod_{\ell=1}^{L} p(b_{1\ell}, \ldots, b_{n_\ell^* \ell} | \mu_{1,\ell}, v_\ell^*) \times p(n_\ell^* | n_\ell^* \geq 1, \; v_\ell^*)$$

$$\times p(v_\ell^* | \mu_{1,\ell}, \Theta) \times p(\mu_{1,\ell} | \beta, \sigma^2) \prod_{i=1}^{n_\ell^*} \mathbb{1}\left[\overline{b}_\ell \geq b_{i\ell} \geq p_{0\ell}\right]$$

MCMC

- Sampling $(\mu_{1,\ell}, v_\ell^*)$ using the M-H algorithm.
  Use a simple random walk proposal density
  $$q(\mu_{1,\ell}^{new}, v_\ell^{*new} | \mu_{1,\ell}^{old}, v_\ell^{*old}) = f_t(\mu_{1,\ell}^{new} | \mu_{1,\ell}^{old}, h_\mu, \omega_\mu)$$
  $\times \frac{f_t(v_\ell^{*new} | v_\ell^{*old}, h_v p_{0\ell}, \omega_v)}{1 - F_t(p_{0\ell} | v_\ell^{*old}, h_v p_{0\ell}, \omega_v)}$
  since v_ℓ^* can only take values between $(p_{0\ell}, \infty)$.
  Moves to the proposal value with probability
  $\alpha\left[\left(\mu_{1,\ell}^{old}, v_\ell^{*old}\right), \left(\mu_{1,\ell}^{new}, v_\ell^{*new}\right)\right] ==$
  $\min\left\{\frac{\pi(\mu_{1,\ell}^{new}, v_\ell^{*new} | b, n, \beta, \Theta, d_\ell, \sigma_\ell^2)\left[1 - F_t(p_{0\ell} | v_\ell^{*old}, p_{0\ell} h_v, \omega_v)\right]}{\pi(\mu_{1,\ell}^{old}, v_\ell^{*old} | b, n, \beta, \Theta, d_\ell, \sigma_\ell^2)\left[1 - F_t(p_{0\ell} | v_\ell^{*new}, p_{0\ell} h_v, \omega_v)\right]}, 1\right\}.$
- Sampling β. Draw β given $\mu_{1,\ell}, \sigma^2$ and it's prior, which is a normal distribution with variance
  $\Lambda = \left(B_0 + \sum_{\ell=1}^{L} \sigma^{-2} x_\ell' x_\ell\right)^{-1}$ and mean
  $\overline{\beta} = \Lambda\left(B_0 \beta_0 + \sum_{\ell=1}^{L} \sigma^{-2} x_\ell' \mu_{1,\ell}\right).$

- Sampling Θ. The full conditional density for Θ is

$$\pi[\Theta|\mu_{1,\ell}, v_\ell^*] = \exp\left[-\left(\Theta-\theta_0\right)' D_0 \left(\Theta-\theta_0\right)/2\right] \times \prod_{\ell=1}^{L} \frac{1}{\exp(\mu_{1,\ell}-x_\ell\Theta)} \exp\left(-\frac{1}{\exp(\mu_{1,\ell}-x_\ell\Theta)} k_\ell\right)$$

  Utilize the M-H algorithm with the proposal density $f_T(\Theta_{new}|\widehat{\Theta}-\left(\Theta_{old}-\widehat{\Theta}\right), \tau V)$ with k degrees of freedom;
  $\widehat{\Theta}$: the mode of $\log \pi[\Theta|\mu_{1,\ell}, v_\ell^*]$;
  V : negative inverse of the Hessian of $\log \pi[\Theta|\mu_{1,\ell}, v_\ell^*]$ evaluated at the modal value $\widehat{\Theta}$;
  k, τ: are tuning paramters;

- Sampling σ^2. Draw σ^2 given $\mu_{1,\ell}, \beta$ and its prior, which is an inverse gamma distribution with parameters $\frac{L+n_0}{2}$ and $\{R_0 +_{\ell=1}^{L} (\mu_{1,\ell}-x_\ell\beta)^2\}/2$.

The Model Selection Problem

- $B_{rs} = \frac{m(y|M_r)}{m(y|M_s)}$;
- Jeffreys scale (evidence against model s): $\log(B_{rs})$ (0, 1.15): not worth a mention; (1.15, 3.45): substantial; (3.45, 4.60): strong; (4.60, ∞): very strong;
- Chib (1995, JASA) notes that by Bayes Theorem

$$m(b, n^*) = \frac{f(b, n^*|\beta, \Theta, \sigma^2) prior(\beta, \Theta, \sigma^2)}{\pi(\beta, \Theta, \sigma^2|b, n^*)}$$

In our case

$$\log \widehat{m}(b, n^*) = \log \widehat{f}(b, n^*|\beta^\#, \Theta^\#, \sigma^{2\#}) + \log \widehat{prior}(\beta^\#, \Theta^\#, \sigma^{2\#}) - \log \widehat{\pi}(\beta^\#, \Theta^\#, \sigma^{2\#}|b, n^*)$$

$\log \widehat{f}(b, n^*|\beta^\#, \Theta^\#, \sigma^{2\#})$: Log likelihood function evaluated at $\left(\beta^\#, \Theta^\#, \sigma^{2\#}\right)$;
$\log \widehat{\pi}(\beta^\#, \Theta^\#, \sigma^{2\#}|b, n^*)$: Posterior ordinate at $\left(\beta^\#, \Theta^\#, \sigma^{2\#}\right)$.

Timber Sales Auction Data

- Collect from the Michigan Department of Natural Resources (MDoNR);
- Focus on data from one regional office—Baldwin;
- Time period: January 1999 to August 2004.
- Auction Mechanism:
  - MDoNR advertises the auctions 4 to 6 weeks prior to the sale date;
  - Each auction has a minimum acceptable bid (the public reserve price);
  - Bids must be submitted before the bid opening time in a sealed envelope (actual bidders).

Summary Statistics

Variable	Obs	Mean	S. D.	Min	Max
Bids	1209	40824.48	36568.68	671.91	229985.70
Winbid	314	42729.71	39666.95	681.90	229985.70
Reserve	332	28205.10	27805.67	601.90	195283.10
Acre	332	72.14	56.55	4	297
Actual	332	3.64	2.30	0	11
Potential	332	12.92	4.90	3	23
Range	332	16.80	8.26	0	66.29
Payment	332	2.36	1.31	1	9
Years	332	2.09	0.21	0.08	3.17

N: approximated by the total number of bidders who submitted an actual bid for any auction held by the same regional office in the same month.

Interesting Feature of the Data

- Endogenous entry:
  - Only 28.17% of the potential bidders actually submit their bids.
  - 5.42% of the auctions receive no bids.

- Evidence supporting Model I:
  - Each lot will not be harvested again in 60 years;
  - Volumn estimation error range given in the ads range from 0% to 66.29%;
  - Hence strong incentives to cruise the lot, which is costly;

- Evidence supporting Model II:
  Most bidders are local logging companies and sawmills.

Estimation results

Table 3 Bayesian Estimation of Structural Models

	Model I		Model II	
Variable	Mean	Stan. Dev.	Mean	Stan. Dev.
log(Reserve)	0.9259**	0.0409	0.9839**	0.0427
Acre	0.0888	0.0632	-0.0934	0.0700
Range	0.1382	0.3608	-0.4138	0.4001
Payments	0.0164	0.0242	0.0136	0.0268
Years	0.0225	0.1197	0.1131	0.1340
Constant	0.5950	0.4479	-0.7385	0.4563
log(Reserve)	0.7624**	0.1640	0.8974**	0.1732
Acre	0.2966	0.2983	-0.0172	0.3475
Range	0.1673	1.4128	-0.4872	1.4289
Payments	0.0450	0.1163	0.0309	0.1263
Years	-0.1248	0.5951	-0.0805	0.6044
Constant	0.5197	1.4267	-1.0304	1.3961
σ^2	0.1069**	0.0160	0.2616**	0.0215

More on the Results from Structural Inference

Table 4 Quantities of Economic Interest

Variable	Model I	Model II
$\frac{\text{Entry Cost}}{\text{Private Value}}$	9.48%	6.84%
$\frac{\text{Entry Cost}}{\text{Reserve Price}}$	16.64%	12.92%
Winner's Payoff	$13949.61	$42544.51
Information Rent	33.01%	100.94%
Private Value	$51026.23	$62253.76

- Model selection result: $\log(B_{21}) = 15252.58$, "very strong" evidence against model I.

Counterfactual Analysis I: Quantifying the Revenue Gain for the Seller from Using the Optimal Reserve Price

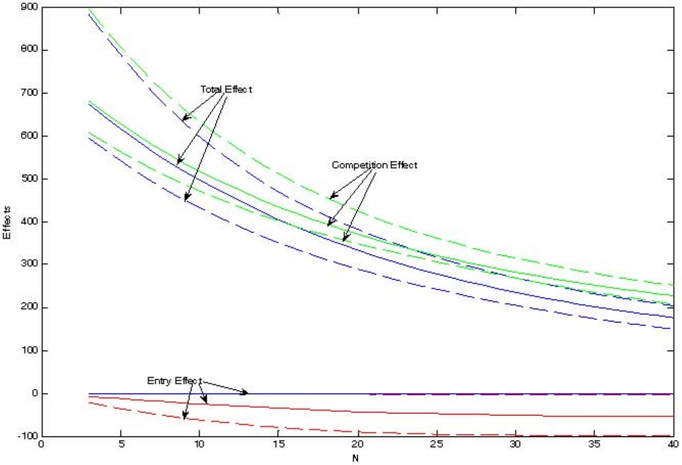
- Set reserve price at

$$p_0^{opt} = v_0 + \frac{1 - F(v^{*opt})}{f(v^{*opt})}$$

  where v^{*opt} is defined implicitly in $v^* = v_0 + \frac{1-F(v^*)}{f(v^*)} + \frac{k}{F(v^*)^{N-1}}$.
- 67 out of 314 auctions goes unsold;
- Total revenue increases 4.7%;
- Average seller's gain is $4,847,357.

Counterfactual Analysis II: Quantifying the “Competition Effect,” “Entry Effect” and “Total Effect” of N on b

- The “competition effect ”is always positive;
- The “entry effect ”is always negative;
- The postive “competition effect ”significantly donimates the negative “entry effect;
- For example, when $N = 5$, the simuated mean of the equilibrium bid for the representative auction is $33,262, while it becomes $33,863 (a 1.8% increase) when $N = 6$.


Total Effect
Competition Effect
Entry Effect
Effects
N
900
800
700
600
500
400
300
200
100
0
-100
0
5
10
15
20
25
30
35
40

Conclusions

- Provide a unified framework for estimating and selecting between two competing entry/bidding model;
- Obtain some new theoretical results for auction models with entry costs;
- Apply the method to analyze timber sale auctions:
    - Seller can gain significantly from using the optimal reserve price;
    - Postive competition effect dominates negative entry effect, hence it is desirable to encourage more competition.